International Journal of Applied Mathematics and Statistical Sciences (IJAMSS)
ISSN (P): 2319-3972; ISSN (E): 2319-3980
Vol. 9, Issue 5, Jul–Dec 2020; 11–20
© IASET

International Academy of Science, Engineering and Technology
IASET Connecting Researchers; Nurturing Innovations

COMPUTATIONAL APPROACH TO RICCATI DIFFERENTIAL EQUATIONS

Andefiki, J, Awari, Y. S & Shallom, D

Research Scholar, Department of Mathematical Sciences, Taraba State University, Jalingo

ABSTRACT

Riccati differential equation is one of the most essential tools for modelling many physical situations, such as spring mass systems, resistor-capacitor-induction circuits, and chemical reactions among many others. It is applicable in engineering and science, and also useful in network synthesis and optimal control. We derived a quarter-step method for the solution of RDEs by collocating and interpolating the Laguerre polynomial basis function which does not require starting values before they are implemented and they simultaneously generate approximations at different grid points in the interval of integration. To show the accuracy and efficiency of our method, five (5) model RDE problems were solved and results obtained in terms of the point wise absolute errors shows that the method approximates well with the exact solution. The stability analysis conducted reveals that our method is zero-stable, consistent and convergent.

KEYWORDS: *Lequerre Polynomial, Blockhybrid Method, Ricatti Differential Equation, Integration Interval, Weight Function*

Article History

Received:* *18 Mar 2020* | *Revised:* *05 Aug 2020* | *Accepted:* *24 Aug 2020

1. INTRODUCTION

We consider a numerical method for solving general Ricatti differential equation of the form

$$y'(t) = a(t) + b(t)y(t) + c(t)y^2(t) \tag{1.1}$$

$$y(t_0) = y_0 \tag{1.2}$$

Where $a(t), b(t), c(t)$ are continuous with $c(t) \neq 0$ and t_0, y_0 are arbitrary constants for $y(t)$ which is an unknown function.

The RDE in (1.1) can also be denoted by the equation below;

$$y'(t) = f(t, y) \tag{1.3}$$

The conventional linear multistep method associated with equation (1.3) is given in the form

$$\sum_{i=0}^{k} \phi_i y_{n+i} = h\sum_{i=0}^{k} \psi_i f_{n+i} \tag{1.4}$$

Where, ϕ_i and ψ_i are unknown coefficients of the method to be uniquely determined.

A differential equation in which the unknown function is a function of two or more independent variable is called a Partial Differential Equations (PDEs). Those in which the unknown function is function of only one independent variable are called Ordinary Deferential Equations (ODEs).

Many scholars have worked extensively on the solution of (1.1) in literatures [1-10,12, 14,15,16 and 17]. These authors proposed different method ranging from predictor corrector method to block method using different polynomials as basis functions, evaluated at some selected points.

In this paper, we proposed three-step quarter block hybrid methodusing Leguere polynomials as a basis function, evaluated at grid and off-grid points to give the needed discrete schemes. These discrete schemes are then combined to form the block hybrid method required for implementation.

2. METHODOLOGY

In this section, we proposed a computational method for the solution of RDEs of the form (1.1). Themethod doesnot require starting values for its implementation and it generates approximations at different grid points in the interval of integration. Another advantage of the method is that it is less expensive in terms of function evaluations when compared to the LMMs or the Runge-Kutta methods. The major idea in this work is to approximate the exact solution $y(x)$ to (1.1) on the partition, $\pi_{[a,b]} = [a = x_0 < x_1 < x_2 < \cdots < x_n < x_{n+1} < \cdots < x_N = b]$ of the quarter-step integration interval $[a, b]$ by Laguerre polynomial basis function given by,

$$y(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}(x^n e^{-x}) \tag{2.1}$$

One of the advantages of the Laguerre polynomial (2.1) is that it is orthogonal with respect to the weight function $w(x) = e^{-x}$ on $[0, \infty)$, Raisinghania (2014). The quarter-step computational method shall be derived using interpolation and collocation procedures. Equation (2.1) is interpolated at a grid point; the first derivative of (2.1) is then substituted into (1.1) to obtain a differential system which is evaluated at all grid points. Using this technique, in the form of linear multistep method, we obtain linear multistep methods which are then put in block form to obtain the new quarter-step computational method.

2.1 Derivation of the Generalized Computational Method

Let the Laguerre polynomial approximate solution be given by a function of a single variable of the form,

$$y(x) = \sum_{n=0}^{r+s-1}\left[\frac{e^x}{n!}\frac{d^n}{dx^n}(x^n e^{-x})\right] = 1 + (x-1) + (x^2 - 4x + 2) + \cdots \tag{2.2}$$

Where r and s are the number of collocation and interpolation points respectively. Now, interpolating (2.2) at point $x_{n+s} = 0$ and collocating its first derivatives at points $x_{n+r} = 0(1)k - 1$, leads to the following system of equations,

$$XA = U \tag{2.3}$$

Where

$$A = [a_0 a_1 a_2 \cdots a_r]^T, \quad U = [y_n f_n f_{n+1} \cdots f_{n+r}]^T$$

$$X = \begin{bmatrix} x_n^0 & x_n^1 & x_n^2 & x_n^3 & \cdots & x_n^N \\ 0 & 1 & 2x_n & 3x_n^2 & \cdots & (N)x_n^{N-1} \\ . & & & & & \\ . & & & & & \\ . & & & & & \\ 0 & 1 & 2x_{n+r} & 3x_{n+r}^2 & \cdots & (N)x_{n+r}^{N-1} \end{bmatrix}$$

Equation (2.3) is then solved for the $a_{j'}s$ using Gaussian elimination method. The values of $a_{j'}s$ obtained are then substituted into (2.2) to give, after some manipulations, a continuous linear multistep method of the form,

$$y(x) = \sum_{j=0}^{s} \alpha_j(x) y_{n+j} + h \sum_{j=0}^{r} \beta_j(x) f_{n+j} \tag{2.4}$$

Where $y_{n+j} = y(x_n + jh)$ and $f_{n+j} = f\left(x_{n+j}, y_{n+j}\right)$. Again, $\alpha_j(x)$and $\beta_j(x)$ are unknown function to be determined and are expressed as continuous functions of t by writing.

$$t = \frac{x - x_n}{h} \tag{2.5}$$

Thus, evaluating (2.4) at $x = x_{n+1}, x_{n+2}, \cdots, x_{n+r}$ using (2.5), we obtain a set of $(r-1)$ single schemes which can be written in the form of discrete quarter-step computational method as,

$$\mathrm{A}^{(0)} Y_m = E Y_n + h d f(y_n) + h b F(Y_m) \tag{2.6}$$

Where

$$Y_m = [y_{n+1}, y_{n+2} \cdots y_{n+r}]^T; \qquad Y_n = [y_{n-(r+1)}, y_{n-(r+2)} \cdots y_n]^T$$

$$F(Y_m) = [f_{n+1}, f_{n+2} \cdots f_{n+r}]^T f(y_n) = [f_{n-(r-1)}, f_{n-(r-2)} \cdots f_n]^T$$

$A^{(0)}, E, d$ and b are $r \times r$ matrices.Equation (2.6) is referred to as the discrete computational method which gives evaluation at different grid points without overlapping.

2.2 Derivation of Computational Method with Three Partitions

Let the approximate solution to (1.1) be given by Laguerre polynomial of degree 5, by allowing $r + s - 1$ in equation 2.2, that is,

$$y(x) = \sum_{n=0}^{5} \left[\frac{e^x}{n!} \frac{d^n}{dx^n} (x^n e^{-x})\right] = 720 - 1800x + 1200x^2 - 300x^3 + 30x^4 - x^5 \tag{2.7}$$

with the first derivative given by,

$$y'(x) = -1800 + 2400x - 900x^2 + 120x^3 - 5x^4 \tag{2.8}$$

Substituting (2.8) into (1.1) gives,

$$f(x, y) = -1800 + 2400x - 900x^2 + 120x^3 - 5x^4 \tag{2.9}$$

Now, interpolating (2.7) at point $x_{n+s} = 0$ and collocating (2.9) at points $x_{n+r}, r = 0\left(\frac{1}{16}\right)\frac{1}{4}$, leads to a system of nonlinear equation of the form (2.3), where

$$A = [a_0\,, a_1\,, a_2\,, a_3\,, a_4\,, a_5]^T$$

$$U = \left[y_n, f_n, f_{n+\frac{1}{16}}, f_{n+\frac{1}{8}}, f_{n+\frac{3}{16}}, f_{n+\frac{1}{4}}\right]^T$$

$$X = \begin{bmatrix} 720 & -1800x_n & 1200x_n^2 & -300x_n^3 & 30x_n^4 & -x_n^5 \\ 0 & -1800 & 2400x_n^2 & -900x_n^2 & 120x_n^3 & -5x_n^4 \\ 0 & -1800 & 2400x_{n+\frac{1}{16}} & -900x^2_{n+\frac{1}{16}} & -120x^3_{n+\frac{1}{16}} & -5x^4_{n+\frac{1}{16}} \\ 0 & -1800 & 2400x_{n+\frac{1}{8}} & -900x^2_{n+\frac{1}{8}} & -120x^3_{n+\frac{1}{8}} & -5x^4_{n+\frac{1}{8}} \\ 0 & -1800 & 2400x_{n+\frac{3}{16}} & -900x^2_{n+\frac{3}{16}} & -120x^3_{n+\frac{3}{16}} & -5x^4_{n+\frac{3}{16}} \\ 0 & -1800 & 2400x_{n+\frac{1}{4}} & -900x^2_{n+\frac{1}{4}} & -120x^3_{n+\frac{1}{4}} & -5x^4_{n+\frac{1}{4}} \end{bmatrix}$$

Which yields the following matrices

$$\begin{pmatrix} 720 & 0 & 0 & 0 & 0 & 0 \\ 0 & -18000 & 0 & 0 & 0 & 0 \\ 0 & -18000 & 150 & -\frac{225}{64} & -\frac{15}{512} & -\frac{5}{65536} \\ 0 & -18000 & 300 & -\frac{225}{16} & \frac{15}{64} & -\frac{5}{4096} \\ 0 & -18000 & 450 & -\frac{2025}{64} & \frac{405}{512} & -\frac{405}{65536} \\ 0 & -18000 & 600 & -\frac{225}{4} & \frac{15}{8} & -\frac{5}{256} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{pmatrix} = \begin{pmatrix} y_n \\ f_n \\ f_{n+\frac{1}{16}} \\ f_{n+\frac{1}{8}} \\ f_{n+\frac{3}{16}} \\ f_{n+\frac{1}{4}} \end{pmatrix}$$

Solving (2.3) by Gaussian elimination method for the $a_j's, j = 0(1)5$ and substituting back into the Laguerre polynomial basis function gives a quarter-step method of the form,

$$y(x) = \alpha_0(x)y_n + h\sum_{j=0}^{\frac{1}{4}} \beta_j(x)f_{n+j}, \; j = 0\left(\frac{1}{16}\right)\frac{1}{4} \tag{2.10}$$

Where the coefficients of y_n and f_{n+j} are give as,

$$\left.\begin{aligned} &\alpha_0 = 1 \\ &\beta_0 = \frac{1}{45}(24576t^5 - 19200t^4 + 5600t^3 - 750t^2 + 45t) \\ &\beta_{\frac{1}{16}} = \frac{32}{45}(-3072t^5 + 2160t^4 - 520t^3 + 45t^2) \\ &\beta_{\frac{1}{8}} = -\frac{8}{15}(-6144t^5 + 3840t^4 - 760t^3 + 45t^2) \\ &\beta_{\frac{3}{16}} = -3072t^5 + 1680t^4 - 280t^3 + 15t^2 \\ &\beta_{\frac{1}{4}} = -\frac{2}{45}(-12288t^5 + 5760t^4 - 880t^3 + 45t^2) \end{aligned}\right\} \tag{2.11}$$

And t is given by (2.5). Evaluating (2.10) at $t = \frac{1}{16}\left(\frac{8}{16}\right)\frac{1}{4}$gives a discrete quarter-step computational method of the form (2.6).

$$\left.\begin{aligned}
y_{n+1} &= y_n + \frac{10271}{45}hf_n + \frac{24152}{15}hf_{n+\frac{1}{8}} - \frac{53024}{45}hf_{n+\frac{3}{16}} + \frac{14726}{45}hf_{n+\frac{1}{4}} - \frac{44384}{45}hf_{n+\frac{1}{16}} \\
y_{n+\frac{1}{16}} &= y_n + \frac{251}{11520}hf_n - \frac{11}{480}hf_{n+\frac{1}{8}} + \frac{53}{5760}hf_{\frac{3}{16}} - \frac{19}{11520}hf_{\frac{1}{4}} + \frac{323}{5760}hf_{n+\frac{1}{16}} \\
y_{n+\frac{1}{8}} &= y_n + \frac{29}{1440}hf_n + \frac{1}{60}hf_{n+\frac{1}{8}} + \frac{1}{360}hf_{n+\frac{3}{16}} - \frac{1}{1440}hf_{n+\frac{1}{4}} + \frac{31}{360}hf_{n+\frac{1}{16}} \\
y_{n+\frac{3}{16}} &= y_n + \frac{27}{1280}hf_n + \frac{9}{160}hf_{n+\frac{1}{8}} + \frac{21}{640}hf_{n+\frac{3}{16}} - \frac{3}{1280}hf_{n+\frac{1}{4}} + \frac{51}{640}hf_{n+\frac{1}{16}} \\
y_{n+\frac{1}{4}} &= y_n + \frac{7}{360}hf_n + 1/30hf_{\frac{1}{8}} + \frac{4}{45}hf_{n+\frac{3}{16}} + \frac{7}{360}hf_{n+\frac{1}{4}} + \frac{4}{45}hf_{n+\frac{1}{16}}
\end{aligned}\right\} \quad (2.12)$$

3. ANALYSIS OF THE METHOD

3.1 Order of the Method

The equation (1.4) associated with lineardifference operator $L$is defined by:

$$L[y(x):h] = \sum_{j=0}^{k} [\alpha_j y(x_n + jh) - h\beta_j y'(x_n + jh)] \quad (3.1)$$

Where $y(x)$ is an arbitrary test function that is continuously differentiable in the interval $[a, b]$. Expanding $y(x_n + jh)$ and $y'(x_n + jh)$ in Taylor series about x_n and collecting like terms in h and $y(x)$ gives:

$$L[y(x):h] = c_0 y(x) + c_1^{(1)} hy'(x) + c_2^{(1)} y(x) + \cdots + c_p h^p y^p(x) \quad (3.2)$$

Accordingly, following [11] and [14], the differential operator and its associated Linear Multistep Method are said to be of order p if:

$$c_0 = c_1 = c_2 = \cdots c_p = c_{p+1} = 0, c_{p+1} \neq 0$$

The term C_{p+1} is called error constant and it implies that the local truncation error is given by

$$E_{n+k} = c_{p+1} h^{p+1} y^{p+1}(x_n) + 0(h^{p+2})$$

3.2 Consistency

In the spirit of [13], the linear multistep method is said to be Consistent if it has order $p \geq 1$. Analysis of our method shows that it is consistent since its order $p = 5 > 1$ (see table 4.6)

3.3 Zero-Stability

The linear Multistep Method is said to be zerostable if no root of the first characteristic polynomial has modulus greater than one and if every root with modulus is simple.

The hybrid block method is said to be stable if the root z of the characteristic polynomial $\bar{p}(z)$, defined by:

$$\rho(R) = \det[RA - A']$$

satisfies $|R| \leq 1$ and every root with $|z_0| = 1$ has multiplicity not exceeding two in the limit as $n \to 0$.

Theorem 1.1 The necessary and sufficient condition for a method to be convergent is for it to be consistent and zero stable

3.4 Stability of the Computational Method

3.4.1 Stability of the Computational Method with Three Partitions

The equation (3.12) when put together formed the block as

$$\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+1}\\y_{n+\frac{1}{16}}\\y_{n+\frac{1}{8}}\\y_{n+\frac{3}{16}}\\y_{n+\frac{1}{4}}\end{bmatrix}=\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n-4}\\y_{n-3}\\y_{n-2}\\y_{n-1}\\y_{n}\end{bmatrix}$$

$$+h\begin{bmatrix}\frac{24152}{15}&\frac{53024}{45}&\frac{14726}{45}&\frac{44384}{45}\\\frac{11}{480}&\frac{53}{5760}&\frac{19}{11520}&\frac{323}{5760}\\\frac{1}{60}&\frac{1}{360}&\frac{1}{1440}&\frac{31}{360}\\\frac{9}{160}&\frac{21}{640}&\frac{3}{1280}&\frac{51}{640}\\\frac{1}{30}&\frac{4}{45}&\frac{7}{360}&\frac{4}{45}\end{bmatrix}\begin{bmatrix}f_{n+1}\\f_{n+\frac{1}{16}}\\f_{n+\frac{1}{8}}\\f_{n+\frac{3}{16}}\\f_{n+\frac{1}{4}}\end{bmatrix}$$

$$+h\begin{bmatrix}0&0&0&0&\frac{10271}{45}\\0&0&0&0&\frac{251}{11520}\\0&0&0&0&\frac{29}{1440}\\0&0&0&0&\frac{27}{1280}\\0&0&0&0&\frac{7}{360}\end{bmatrix}\begin{bmatrix}f_{n-4}\\f_{n-3}\\f_{n-2}\\f_{n-1}\\f_{n}\end{bmatrix}$$

normalizing the matrix as $h \to 0$

$$\rho(Z) = [ZA^0 - A]$$

$$\begin{bmatrix}Z&0&0&0&0\\0&Z&0&0&0\\0&0&Z&0&0\\0&0&0&Z&0\\0&0&0&0&Z\end{bmatrix}-\begin{bmatrix}0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&0&0&0&1\end{bmatrix}=\begin{bmatrix}Z&0&0&0&-1\\0&Z&0&0&-1\\0&0&Z&0&-1\\0&0&0&Z&-1\\0&0&0&Z&-1\end{bmatrix}$$

$$Z^4(Z-1)=0$$

That is $Z_1 = 1,\ Z_2 = Z_3 = Z_4 = Z_5 = 0$

3.5 Convergence

ALMM is said to be convergent if it is consistent and zero stable.

From the analysis on zero stability above and in addition to table 4.6, we conclude that our method is both consistent and zero stable, hence convergent.

4. NUMERICAL EXAMPLES

The computational method derived shall be applied to some modeled RDEs to test for accuracy and efficiency.

Problem 4.1

Consider the following quadratic Riccati differential equation (RDE).

$$\left(\frac{dy}{dt} = 2y(t) - t^2(t) + 1\right)\frac{dy}{dt} = -y^2(t) + 1$$

subject to the initial condition

$$y(0) = 0$$

with theoreticalsolution given as

$$y(t) = \frac{e^{2t} - 1}{e^{2t} + 1}$$

Problem 4.2

Given the RDE

$$\frac{dy}{dt} = 2y(t) - y^2 + 1$$

with initial condition

$$y(0) = 0$$

and theoretical solution obtained thus,

$$y(t) = 1 + \sqrt{2}\tanh\left(\sqrt{2}t + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right)$$

Problem 4.3

Consider the following nonlinear fractional Riccati differential equation

$$D^{\propto}y(t) = 1 + 2y(t) - y^2(t), 0 < 0n1$$

and initial condition

$$y(0) = 0$$

the theoretical solution for $\propto = 1$ was found to be

$$y(t) = 1 + \sqrt{2}\tanh(\sqrt{2t} + \frac{1}{2}log\frac{\sqrt{2}-1}{\sqrt{2}+1}),$$

Problem 4.4

Let us consider the problem

$$\begin{cases} u'(x) = 1 - u^2(x), 0 \le x \le 1; \\ u(0) = 0. \end{cases},$$

The theoretical solution is

$$U(x) = \frac{e^{2x} - 1}{e^{2x} + 1},$$

Problem 4.5

Consider a Riccati differential equation with constant coefficients gien as

$$y'(t) = y^2(t) - y(t), \quad y(0) = 0,$$

The theoretical solution for this problem is

$$y = \frac{e^{-t}}{1 + e^{-t}},$$

Table 4.1: Numerical Results for Problem 4.1

t-value	Theoretical Solution	Approximate	Abs Error	Time
0.1000	0.42188080599029	0.42188080599029	1.6653e-16	0.0358
0.2000	0.65507140458179	0.65507140458179	3.3307e-16	0.0495
0.3000	0.84956591586548	0.84956591586548	3.3307e-16	0.0631
0.4000	1.01845953531826	1.01845953531826	4.4409e-16	0.0736
0.5000	1.16698055323955	1.16698055323955	4.4409e-16	0.0865
0.6000	1.29820152510130	1.29820152510130	6.6613e-16	0.0950
0.7000	1.41436135877797	1.41436135877797	6.6613e-16	0.1028
0.8000	1.51728483742558	1.51728483742558	6.6613e-16	0.1109
0.9000	1.60853886960997	1.60853886960997	6.6613e-16	0.1207
1.0000	1.68949839159438	1.68949839159438	6.6613e-16	0.1275

Table 4.2: Numerical Results for Problem 4.2

t-value	Theoretical Solution	Approximate	Abs Error	Time
0.1000	0.09966799462496	0.09967169539817	3.7008e-06	0.0418
0.2000	0.19737532022490	0.19738963167954	1.4311e-05	0.0543
0.3000	0.29131261245159	0.29134303769860	3.0425e-05	0.0680
0.4000	0.37994896225523	0.37999898473952	5.0022e-05	0.0803
0.5000	0.46211715726001	0.46218801205332	7.0855e-05	0.0907
0.6000	0.53704956699804	0.53714038396459	9.0817e-05	0.1301
0.7000	0.60436777711716	0.60447600054929	1.0822e-04	0.1404
0.8000	0.66403677026785	0.66415872119655	1.2195e-04	0.1504
0.9000	0.71629787019902	0.71642932361034	1.3145e-04	0.1603
1.0000	0.76159415595577	0.76173083991998	1.3668e-04	0.1697

Table 4.3: Numerical Results for Problem 4.3

t-value	Theoretical Solution	Approximate	Abs Error	Time
0.1000	0.90909090909091	0.90902868973486	6.2219e-05	0.0361
0.2000	0.83333333333333	0.83322783539319	1.0550e-04	0.0459
0.3000	0.76923076923077	0.76909402754165	1.3674e-04	0.0576
0.4000	0.71428571428571	0.71412555961818	1.6015e-04	0.0646
0.5000	0.66666666666667	0.66648827625894	1.7839e-04	0.0746
0.6000	0.62500000000000	0.62480682274283	1.9318e-04	0.0853
0.7000	0.58823529411765	0.58802962114848	2.0567e-04	0.0950
0.8000	0.55555555555556	0.55533888221389	2.1667e-04	0.1048
0.9000	0.52631578947368	0.52608905048065	2.2674e-04	0.1145
1.0000	0.50000000000000	0.49976372539295	2.3627e-04	0.1223

Table 4.4: Numerical Results for Problem 4.4

t-value	Theoretical Solution	Approximate	Abs Error	Time
0.1000	**1.12295995501999**	1.12295995501999	5.6155e-13	0.0369
0.2000	**2.33036366723934**	2.33036366723934	1.1653e-12	0.0506
0.3000	3.35929859139219	3.35929859139219	1.6800e-12	0.0642
0.4000	4.07625619989395	4.07625619989395	2.0384e-12	0.0750
0.5000	4.50864023794231	4.50864023794231	2.2542e-12	0.0876
0.6000	4.74705986375187	4.74705986375187	2.3741e-12	0.0972
0.7000	4.87206646548955	4.87206646548955	2.4363e-12	0.1066
0.8000	4.93588015111826	4.93588015111826	2.4682e-12	0.1160
0.9000	4.96801151790818	4.96801151790818	2.4842e-12	0.1268
1.0000	4.98407836223864	4.98407836223864	2.4922e-12	0.1343

Table 4.5: Numerical Results for Problem 4.5

t-value	Theoretical Solution	Approximate	Abs Error	Time
0.1000	-0.09966799462496	-0.09966799462496	2.1917e-010	0.0307
0.2000	**-0.19737532022490**	-0.19737532022490	1.6820e-009	0.0423
0.3000	-0.29131261245159	-0.29131261245159	5.3018e-009	0.0549
0.4000	-0.37994896225523	-0.37994896225523	1.1443e-008	0.0661
0.5000	**-0.46211715726001**	-0.46211715726001	1.9872e-008	0.0754
0.6000	-0.53704956699804	-0.53704956699804	2.9881e-008	0.0852
0.7000	-0.60436777711716	-0.60436777711716	4.0496e-008	0.0937
0.8000	-0.66403677026785	-0.66403677026785	5.0713e-008	0.1008
0.9000	-0.71629787019902	-0.71629787019902	5.9682e-008	0.1064
1.0000	-0.76159415595577	-0.76159415595577	6.6810e-008	0.1159

Table 4.6: Order and Error Constants of the New Computational Method

Point of Evaluation	c_{p+1}	Error Constant
x_{n+1}	5	$\frac{393}{655360}$
$x_{n+\frac{1}{16}}$	5	$\frac{3}{2684354560}$
$x_{n+\frac{1}{8}}$	5	$\frac{1}{1509949440}$
$x_{n+\frac{3}{16}}$	5	$\frac{3}{2684354560}$
$x_{n+\frac{1}{4}}$	5	$-\frac{1}{31708938240}$

5. CONCLUTIONS

The desirable property of a numerical solution is to behave like the theoretical solution, that is, a good numerical solution is one which always converges to its theoretical solution as $h \to 0$. This is an essential property that all numerical methods should possess. We are therefore very confident in presenting our results in table 4.1 to table 4.5. In all the examples provided, our method tends to converge to its theoretical solution faster, making it one of the favorable methods to be considered in solving real life problems. We therefore recommend our method to the scientific world for further investigation and application.

REFERENCES

1. *Abbasbandy S. (2006). Homotopy perturbation method for quadratic Riccati differential equation and comparism with Adomian's decomposition method. Applied Mathematics and Computation, 172(1): 485-490.*

2. *Abdulaziz, O., Noor, N. F. M., Hashim, I., Noorani, M. S. M. (2008). Further Accuracy Tests on Accuracy of the Adomian Decomposition Method for Chaotic Systems. Chaos, Solitons and Fractals, 36(5), 1405-1411. https://doi.org/10.1016/j.chaos.2006.09.007*

3. *Abiodun A. Opanuga, Sunday O. Edeki1, Hilary I. Okabgue, and GraceO. Akinlabi (2015). A novel approach for solving Riccati differential equation. International Journal of Applied Engineering Research, Volume 10 , Issue 11 , Pages 29121-29126*

4. *Anake, T. A. (2011). Continuous Implicit Hybrid One-Step Methods for the solution of Initial Value Problems of General Second-Order Ordinary Differential Equations. eprint.convenantuniversity.edu.ng>CUGP060192-Anake*

5. *Anderson, B. D. & Moore, J. B. (1999). Optimal Control Linear Quadratic Methods. Prentice- Appl. Math. Comput., 114: 115-123.*

6. *Arikoglu, A. & Ozkol, I. (2007). Solution of Fractional Differential Equations by using differential transform method. Applied Mathematics and Computation, vol. 181, no.1 pp. 153-162*

7. *Balaji, S. (2014). Solution of nonlinear Riccati differential equations using Chebyshev wavelets. Sastra University. WSEAS Transactions on Mathematics 13: 274*

8. *Biazar, J., Eslami, M. (2010). Differential Transform Method for Quadratic Riccati Differential Equation. Universty of Guilan. 1427*

9. *Bildik, N. & Deniz, S. (2015). Modified Adomian Decompositon Method for Solving RDEs. Celal University, Manisa/Turkey. Review of the Air Force Academy No.3(30)*

10. *Edmond Laguerre (1834-1886). Generalized Laguerre polynomials. Springer, New York, NY. An Atlas of Functions pp 209-216*

11. *Fatunla,S.O., (1991). Block methods for second order IVP's, Inter.J.Comp.Maths. 41: 55-63*

12. *File G, Aya T. (2016). Numerical solution of quadratic Riccati differential equations. Egyptian Journal of Basic and Applied Sciences. 3:392-397.*

13. *Henrici,P., (1962). Discrete variable methods for ODE's. John Wiley, New York, 1962*

14. *Lambert, J. D.(1973): Computational Methods in Ordinary Differential Equations.(Introductory Mathematics for Scientists and Engineers). Journal of Applied Mathematics and Mechanics. John Willey and sons. Vol.54, Issue 7*

15. *Riaz, S., Rafiq, M., Ahmad, O. (2015). Nonstandard Finite Difference Method for Quadratic Riccati Differential Equations. Pakistan Punjab University J. Math., 47(2): 1-10*

16. *Tan Y, Abbasbandy S. (2008): Homotopy analysis method for quadratic Riccati differential equations. Commun. Nonlinear Sci. Numer. Simul.;13:539-546.*

17. *Vahidi AR, Didgar M. (2012): Improving the accuracy of the solutions of Riccati equations. Intern. J. Ind. Math.4(1): 11-20.*